Interactive Proofs in Higher-Order Concurrent Separation Logic

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Goal of this talk

Many POPL papers about complicated program logics come with mechanized soundness proofs, but how to reason in these logics?

Goal: reasoning in an object logic in the same style as reasoning in Coq

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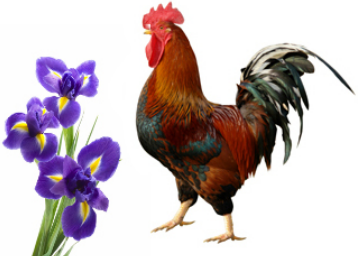
How?

- Extend Coq with (spatial and non-spatial) named proof contexts for an object logic
- Tactics for introduction and elimination of all connectives of the object logic
- Entirely implemented using reflection, type classes and Ltac (no OCaml plugin needed)



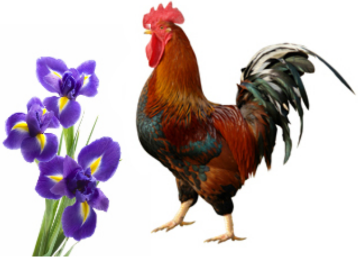
Goal of this talk

Many POPL papers about complicated program logics come with mechanized soundness proofs, but how to reason in these logics?

Goal: reasoning in Iris in the same style as reasoning in Coq

How?

- Extend Coq with (spatial and non-spatial) named proof contexts for Iris
- Tactics for introduction and elimination of all connectives of Iris
- Entirely implemented using reflection, type classes and Ltac (no OCaml plugin needed)



Iris: language independent higher-order separation logic for modular reasoning about fine-grained concurrency in Coq

Iris Proof Mode (IPM) demo

```
Lemma and_exist_sep {A} P R (Ψ: A → iProp) :
  P ∗ (∃ a, Ψ a) ∗ R −∗ ∃ a, Ψ a ∗ P.
Proof.
```

```
1 subgoal
M : ucmraT
A : Type
P, R : iProp
Ψ : A → iProp
______________________________________(1/1)
P ∗ (∃ a : A, Ψ a) ∗ R −∗ ∃ a : A, Ψ a ∗ P
```

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  iIntros "[HP [HΨ HR]]".
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```
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M : ucmraT
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______________________________________(1/1)
"HP" : P
"HΨ" : ∃ a : A, Ψ a
"HR" : R
--------------------------------------∗
∃ a : A, Ψ a ∗ P
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Proof.
  iIntros "[HP [HΨ HR]]".
  iDestruct "HΨ" as (x) "HΨ".
  iExists x.
```

```
1 subgoal
M : ucmraT
A : Type
P, R : iProp
Ψ : A → iProp
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______________________________________(1/1)
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  iExists x.
  iSplitL "HΨ".
```

```
1 subgoal
M : ucmraT
A : Type
P, R : iProp
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x : A
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Proof.
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  iDestruct "HΨ" as (x) "HΨ".
  iExists x.
  iSplitL "HΨ".
```

```
2 subgoals
M : ucmraT
A : Type
P, R : iProp
Ψ : A → iProp
x : A
______________________________________(1/2)
"HΨ" : Ψ x
--------------------------------------∗
Ψ x

______________________________________(2/2)
"HP" : P
"HR" : R
--------------------------------------∗
P
```

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Proof.
  iIntros "[HP [HΨ HR]]".
  iDestruct "HΨ" as (x) "HΨ".
  iExists x.
  iSplitL "HΨ".
  - iAssumption.
```

```
1 subgoal
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```

```
This subproof is complete, but there are some unfocused goals:

______________________________________(1/1)
"HP" : P
"HR" : R
--------------------------------------∗
P
```

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1 subgoal
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```

```
No more subgoals.
```

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  - iAssumption.
Qed.
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No more subgoals.
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  iIntros "[HP [HΨ HR]]".
```

Logical notations overridden in scope for Iris

```
  - iAssumption.
  - iAssumption.
Qed.
```

Iris Proof Mode (IPM) demo

```
Lemma and_exist_sep {A} P R (Ψ: A → iProp) :
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Notation for deeply embedded context

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Proof.
  iIntros "[HP [HΨ HR]]".
  Unset Printing Notations.
```

```
1 subgoal
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Proof.
  iIntros "[HP [HΨ HR]]".
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```

```
1 subgoal
M : ucmraT
A : Type@{Top.105}
P, R : uPred M
Ψ : forall _ : A, uPred M
______________________________________(1/1)
@uPred_entails M
 (@of_envs M
   (@Envs M (@Enil (uPred M))
    (@Esnoc (uPred M)
      (@Esnoc (uPred M)
       (@Esnoc (uPred M) (@Enil (uPred M))
         (String
          (Ascii false false false true false false true
            false)
          (String
            (Ascii false false false false true false true
             false) EmptyString)) P)
       (String
         (Ascii false false false true false false true false)
         (String
          (Ascii false true true true false false true true)
          (String
            (Ascii false false false true false true false
             true) EmptyString)))
       (@uPred_exist M A (fun a : A => Ψ a)))
      (String
       (Ascii false false false true false false true false)
```

Motivation

Why should we care about interactive proofs? Why not automate everything?

Infeasible to automate everything, for example:

- Concurrent algorithms in Iris (Jung, Krebbers, Swasey, Timany)
- The Rust type system in Iris (Jung, Jourdan, Dreyer, Krebbers)
- Logical relations in Iris (Krogh-Jespersen, Svendsen, Timany, Birkedal, Tassarotti, Jung, Krebbers)
- Weak memory concurrency in Iris (Kaiser, Dang, Dreyer, Lahav, Vafeiadis)
- Object calculi in Iris (Swasey, Dreyer, Garg)
- Logical atomicity in Iris (Krogh-Jespersen, Zhang, Jung)
- Defining Iris in Iris (Krebbers, Jung, Jourdan, Bizjak, Dreyer, Birkedal)

Most of these projects are formalized in IPM

How to do such proofs in a proof assistant?

Current proof assistant support is limited to basic separation logic:

- **Macros for manipulating Hoare triples:** Appel, Wright, Charge!, ...
- **Heavy automation:** Bedrock, Rtac, ...

Iris has many complicated connectives that are beyond basic separation logic

How to embed a logic into a proof assistant

Deep embedding

```
Inductive form : Type :=
  | iAnd: form → form → form
  | iForall: string → form → form → form
```

Shallow embedding

```
Definition iProp : Type :=
  (* predicates over states *).
Definition iAnd : iProp → iProp → iProp :=
  (* semantic interpretation *).
Definition iForall : ∀ A, (A → iProp) → iProp :=
  (* semantic interpretation *).
```

How to embed a logic into a proof assistant

Deep embedding	**Shallow embedding**

`Inductive form : Type :=`
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	`  (* semantic interpretation *).`

Traverse formulas using Coq functions (fast)	Traverse formulas on the meta level (slow)
Reflective tactics (fast)	Tactics on the meta level (slow)

How to embed a logic into a proof assistant

Deep embedding	Shallow embedding

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	`Definition iProp : Type :=`
	`  (* predicates over states *).`
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	`  (* semantic interpretation *).`
	`Definition iForall : ∀ A, (A → iProp) → iProp :=`
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Traverse formulas using Coq functions (fast)	Traverse formulas on the meta level (slow)
Reflective tactics (fast)	Tactics on the meta level (slow)
Need to explicitly encode binders	Reuse binders of Coq
Need to embed features like lists	Piggy-back on features like lists from Coq

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Grammar of formulas fixed once and forall	Easily extensible with new connectives

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Context manipulation is the prime task of tactics:
Deeply embed contexts, shallowly embed the logic

Deeply embedded contexts in IPM

Visible goal in IPM:

$\vec{\mathtt{x}} : \vec{\phi}$ Variables and pure Coq hypotheses

\-

$\vec{\mathtt{H}}\mathtt{persistent} : \vec{P}$ Persistent hypotheses in object logic

$\overline{}\Box$

$\vec{\mathtt{H}}\mathtt{spatial} : \vec{Q}$ Spatial hypotheses in object logic

$\overline{}*$

R Goal in object logic

Deeply embedded contexts in IPM

Visible goal in IPM:

Propositions that enjoy $P \Leftrightarrow P * P$

$\vec{\texttt{x}} : \vec{\phi}$ Variables and pure Coq hypotheses

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---$\square$

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---$*$

R Goal in object logic

Actual Coq goal (without pretty printing):

$\vec{x}_i : \vec{\phi}_i$

---

of_envs (Envs ... ...) $\vdash R$

where:

```
Record envs :=
  Envs { env_persistent : env iProp; env_spatial : env iProp }.
Coercion of_envs (Δ : envs) : iProp :=
  (⌜ envs_wf Δ ⌝ ∗ □ [∗] env_persistent Δ ∗ [∗] env_spatial Δ)%I.
```

Deeply embedded contexts in IPM

Visible goal in IPM:

Propositions that enjoy $P \Leftrightarrow P * P$

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R Goal in object logic

Actual Coq goal (without pretty printing):

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———————————————————————————

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where:

Association list of shallowly embedded propositions

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Deeply embedded contexts in IPM

Visible goal in IPM:

Propositions that enjoy $P \Leftrightarrow P * P$

$\vec{\mathtt{x}}$: $\vec{\phi}$ Variables and pure Coq hypotheses

$\overline{}$

$\vec{\mathtt{H}}\mathtt{persistent}$: $\vec{P}$ Persistent hypotheses in object logic

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Actual Coq goal (without pretty printing):

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where:

Association list of shallowly embedded propositions

Folded separating conjunction

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The iSplit tactic

```
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Proof.
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"HΨ" : Ψ x
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Ψ x

______________________________________(2/2)
"HP" : P
"HR" : R
--------------------------------------∗
P
```

Implementation of the `iSplit` tactic

Tactics implemented by reflection as mere lemmas:

```
Lemma tac_sep_split Δ Δ₁ Δ₂ lr js Q1 Q2 :
  envs_split lr js Δ = Some (Δ₁,Δ₂) →
  (Δ₁ ⊢ Q1) → (Δ₂ ⊢ Q2) → Δ ⊢ Q1 ∗ Q2.
```

Implementation of the iSplit tactic

Tactics implemented by reflection as mere lemmas:

```
Lemma tac_sep_split Δ Δ₁ Δ₂ lr js Q1 Q2 :
  envs_split lr js Δ = Some (Δ₁,Δ₂) →
  (Δ₁ ⊢ Q1) → (Δ₂ ⊢ Q2) → Δ ⊢ Q1 ∗ Q2.
```

Context splitting implemented as a computable Coq function

Implementation of the iSplit tactic

Tactics implemented by reflection as mere lemmas:

```
Lemma tac_sep_split Δ Δ1 Δ2 lr js Q1 Q2 :
  envs_split lr js Δ = Some (Δ1,Δ2) →
  (Δ1 ⊢ Q1) → (Δ2 ⊢ Q2) → Δ ⊢ Q1 ∗ Q2.
```

Context splitting implemented as a computable Coq function

Ltac wrappers around the reflective tactic:

```
Tactic Notation "iSplitL" constr(Hs) :=
  let Hs := words Hs in
  eapply tac_sep_split with _ _ false Hs _ _;
    [env_cbv; reflexivity ||
      fail "iSplitL: hypotheses" Hs "not found in the context"
    | (* goal 1 *)
    | (* goal 2 *) ].
```

Implementation of the iSplit tactic

Tactics implemented by reflection as mere lemmas:

```
Lemma tac_sep_split Δ Δ₁ Δ₂ lr js Q1 Q2 :
  envs_split lr js Δ = Some (Δ₁,Δ₂) →
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    | (* goal 2 *) ].
```

Report sensible error to the user

The iFrame tactic

```
Lemma and_exist_sep {A} P R (Ψ: A → iProp) :
  P ∗ (∃ a, Ψ a) ∗ R −∗ ∃ a, Ψ a ∗ P.
Proof.
  iIntros "[HP [HΨ HR]]".
  iDestruct "HΨ" as (x) "HΨ".
```

```
1 subgoal
M : ucmraT
A : Type
P, R : iProp
Ψ : A → iProp
x : A
______________________________________(1/1)
"HP" : P
"HΨ" : Ψ x
"HR" : R
--------------------------------------∗
∃ a : A, Ψ a ∗ P
```

The iFrame tactic

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Lemma and_exist_sep {A} P R (Ψ: A → iProp) :
  P ∗ (∃ a, Ψ a) ∗ R −∗ ∃ a, Ψ a ∗ P.
Proof.
  iIntros "[HP [HΨ HR]]".
  iDestruct "HΨ" as (x) "HΨ".
  iFrame "HP".
```

```
1 subgoal
M : ucmraT
A : Type
P, R : iProp
Ψ : A → iProp
x : A
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"HP" : P
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A : Type
P, R : iProp
Ψ : A → iProp
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"HΨ" : Ψ x
"HR" : R
--------------------------------------∗
∃ a : A, Ψ a
```

Implementation of the `iFrame` tactic

Problem: the goal is not deeply embedded, how to manipulate it?

Implementation of the `iFrame` tactic

Problem: the goal is not deeply embedded, how to manipulate it?

Solution: logic programming using type classes

The lemma corresponding to the tactic in Coq:

```
Class Frame (R P Q : iProp) := frame : R ∗ Q ⊢ P.
```

What we want to frame

Conclusion of the new goal in which R is framed

Initial conclusion

```
Lemma tac_frame Δ Δ' i p R P Q :
  envs_lookup_delete i Δ = Some (p, R, Δ') →
  Frame R P Q →
  ((if p then Δ else Δ') ⊢ Q) → Δ ⊢ P.
```

Implementation of the iFrame tactic

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  Frame R P Q →
  ((if p then Δ else Δ') ⊢ Q) → Δ ⊢ P.
```

Note: we support framing under binders (∃, ∀, . . .) and user defined connectives

Implementation of the `iFrame` tactic (2)

Consider the type class:

```
Class Frame (R P Q : iProp) := frame : R ∗ Q ⊢ P.
```

What we want to frame

Conclusion of the new goal in which `R` is framed

Initial conclusion

Implementation of the `iFrame` tactic (2)

Consider the type class:

```
Class Frame (R P Q : iProp) := frame : R ∗ Q ⊢ P.
```

What we want to frame

Initial conclusion

Conclusion of the new goal in which `R` is framed

Instances (rules of the logic program):

```
Instance frame_here R : Frame R R True.
Instance frame_sep_l R P₁ P₂ Q :
  Frame R P₁ Q → Frame R (P₁ ∗ P₂) (Q ∗ P₂).
Instance frame_sep_r R P₁ P₂ Q :
  Frame R P₂ Q → Frame R (P₁ ∗ P₂) (P₁ ∗ Q).
```

Implementation of the `iFrame` tactic (2)

Consider the type class:

```
Class Frame (R P Q : iProp) := frame : R ∗ Q ⊢ P.
```

What we want to frame

Conclusion of the new goal in which R is framed

Initial conclusion

Instances (rules of the logic program):

```
Class MakeSep P Q PQ := make_sep : P ∗ Q ⊣⊢ PQ.
Instance frame_here R : Frame R R True.
Instance frame_sep_l R P₁ P₂ Q Q' :
  Frame R P₁ Q → MakeSep Q P₂ Q' → Frame R (P₁ ∗ P₂) Q'.
Instance frame_sep_r R P₁ P₂ Q Q' :
  Frame R P₂ Q → MakeSep P₁ Q Q' → Frame R (P₁ ∗ P₂) Q'.

Instance make_sep_true_l P : MakeSep True P P | 1.
Instance make_sep_true_r P : MakeSep P True P | 1.
Instance make_sep_default P Q : MakeSep P Q (P ∗ Q) | 2.
```

Proving Hoare triples

Consider:

$$\{x \mapsto v_1 * y \mapsto v_2\} swap(x, y) \{x \mapsto v_2 * y \mapsto v_1\}$$

How to use IPM to manipulate the precondition?

Proving Hoare triples

Consider:

$$\{x \mapsto v_1 * y \mapsto v_2\} swap(x, y) \{x \mapsto v_2 * y \mapsto v_1\}$$

How to use IPM to manipulate the precondition?

Solution: define Hoare triple in terms of weakest preconditions

We let:

$$\{P\}\, e\, \{Q\} \triangleq \square(P \mathrel{-\!\!*} \mathrm{wp}\; e\, \{Q\})$$

where $\mathrm{wp}\; e\, \{Q\}$ gives the *weakest precondition* under which:

- all executions of e are safe
- the final state of e satisfies the postcondition Q

Proving swap using symbolic execution

```
Definition swap : val := λ: "x" "y",
  let: "tmp" := !"x" in
  "x" ← !"y";;
  "y" ← "tmp".

Lemma swap_spec l1 l2 v1 v2 :
  {{ l1 ↦ v1 ∗ l2 ↦ v2 }} swap #l1 #l2
  {{ _, l1 ↦ v2 ∗ l2 ↦ v1 }}.
Proof.
```

```
1 subgoal
Σ : gFunctors
H : heapG Σ
l1, l2 : loc
v1, v2 : val
______________________________________(1/1)
{{ l1 ↦ v1 ∗ l2 ↦ v2 }} (swap #l1) #l2 {{ _, l1 ↦ v2 ∗ l2 ↦ v1 }}
```

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Definition swap : val := λ: "x" "y",
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  "x" ← !"y";;
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Lemma swap_spec l1 l2 v1 v2 :
  {{ l1 ↦ v1 ∗ l2 ↦ v2 }} swap #l1 #l2
  {{ _, l1 ↦ v2 ∗ l2 ↦ v1 }}.
Proof.
  iIntros "!# [Hl1 Hl2]".
```

```
1 subgoal
Σ : gFunctors
H : heapG Σ
l1, l2 : loc
v1, v2 : val
______________________________________(1/1)
"Hl1" : l1 ↦ v1
"Hl2" : l2 ↦ v2
--------------------------------------∗
WP (swap #l1) #l2 {{ _, l1 ↦ v2 ∗ l2 ↦ v1 }}
```

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Definition swap : val := λ: "x" "y",
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  "x" ← !"y";;
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  {{ l1 ↦ v1 ∗ l2 ↦ v2 }} swap #l1 #l2
  {{ _, l1 ↦ v2 ∗ l2 ↦ v1 }}.
Proof.
  iIntros "!# [Hl1 Hl2]".
  do 2 wp_let.
```

```
1 subgoal
Σ : gFunctors
H : heapG Σ
l1, l2 : loc
v1, v2 : val
______________________________________(1/1)
"Hl1" : l1 ↦ v1
"Hl2" : l2 ↦ v2
--------------------------------------∗
WP
  let: "tmp" := ! #l1 in
  #l1 ← ! #l2 ;;
  #l2 ← "tmp" {{ _, l1 ↦ v2 ∗ l2 ↦ v1 }}
```

Proving swap using symbolic execution

```
Definition swap : val := λ: "x" "y",
  let: "tmp" := !"x" in
  "x" ← !"y";;
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Lemma swap_spec l1 l2 v1 v2 :
  {{ l1 ↦ v1 ∗ l2 ↦ v2 }} swap #l1 #l2
  {{ _, l1 ↦ v2 ∗ l2 ↦ v1 }}.
Proof.
  iIntros "!# [Hl1 Hl2]".
  do 2 wp_let.
  wp_load; wp_let.
```

```
1 subgoal
Σ : gFunctors
H : heapG Σ
l1, l2 : loc
v1, v2 : val
____________________________________(1/1)
"Hl1" : l1 ↦ v1
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-------------------------------------∗
WP #l1 ← ! #l2 ;; #l2 ← v1 {{ _, l1 ↦ v2 ∗ l2 ↦ v1 }}
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  do 2 wp_let.
  wp_load; wp_let.
  wp_load.
```

```
1 subgoal
Σ : gFunctors
H : heapG Σ
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v1, v2 : val
______________________________________(1/1)
"Hl1" : l1 ↦ v1
"Hl2" : l2 ↦ v2
--------------------------------------∗
WP #l1 ← v2 ;; #l2 ← v1 {{ _, l1 ↦ v2 ∗ l2 ↦ v1 }}
```

Proving swap using symbolic execution

```
Definition swap : val := λ: "x" "y",
  let: "tmp" := !"x" in
  "x" ← !"y";;
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Lemma swap_spec l1 l2 v1 v2 :
  {{ l1 ↦ v1 ∗ l2 ↦ v2 }} swap #l1 #l2
  {{ _, l1 ↦ v2 ∗ l2 ↦ v1 }}.
Proof.
  iIntros "!# [Hl1 Hl2]".
  do 2 wp_let.
  wp_load; wp_let.
  wp_load.
  wp_store.
```

```
1 subgoal
Σ : gFunctors
H : heapG Σ
l1, l2 : loc
v1, v2 : val
______________________________________(1/1)
"Hl1" : l1 ↦ v2
"Hl2" : l2 ↦ v2
--------------------------------------∗
WP #l2 ← v1 {{ _, l1 ↦ v2 ∗ l2 ↦ v1 }}
```

Proving swap using symbolic execution

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Definition swap : val := λ: "x" "y",
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"Hl1" : l1 ↦ v2
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--------------------------------------∗
l1 ↦ v2 ∗ l2 ↦ v1
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Proving swap using symbolic execution

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Definition swap : val := λ: "x" "y",
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  {{ l1 ↦ v1 ∗ l2 ↦ v2 }} swap #l1 #l2
  {{ _, l1 ↦ v2 ∗ l2 ↦ v1 }}.
Proof.
  iIntros "!# [Hl1 Hl2]".
  do 2 wp_let.
  wp_load; wp_let.
  wp_load.
  wp_store.
  wp_store.
  iFrame.
```

```
No more subgoals.
```

Proving swap using symbolic execution

```
Definition swap : val := λ: "x" "y",
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  {{ _, l1 ↦ v2 ∗ l2 ↦ v1 }}.
Proof.
  iIntros "!# [Hl1 Hl2]".
  do 2 wp_let.
  wp_load; wp_let.
  wp_load.
  wp_store.
  wp_store.
  iFrame.
Qed.
```

Making IPM tactics modular using type classes

We want `iDestruct "H" as "[H1 H2]"` to:

- turn $\texttt{H} : \texttt{P} * \texttt{Q}$ into $\texttt{H1} : \texttt{P}$ and $\texttt{H2} : \texttt{Q}$
- turn $\texttt{H} : \triangleright(\texttt{P} * \texttt{Q})$ into $\texttt{H2} : \triangleright\ \texttt{P}$ and $\texttt{H2} : \triangleright\ \texttt{Q}$
- turn $\texttt{H} : \texttt{l} \mapsto \texttt{v}$ into $\texttt{H1} : \texttt{l} \xmapsto{1/2} \texttt{v}$ and $\texttt{H2} : \texttt{l} \xmapsto{1/2} \texttt{v}$

Making IPM tactics modular using type classes

We want `iDestruct "H" as "[H1 H2]"` to:

- turn `H : P ∗ Q` into `H1 : P` and `H2 : Q`
- turn `H : ▷(P ∗ Q)` into `H2 : ▷ P` and `H2 : ▷ Q`
- turn `H : l ↦ v` into $\texttt{H1 : l} \overset{1/2}{\longmapsto} \texttt{v}$ and $\texttt{H2 : l} \overset{1/2}{\longmapsto} \texttt{v}$

We use type classes to achieve that:

```
Class IntoAnd (p : bool) (P Q1 Q2 : uPred M) :=
  into_and : P ⊢ if p then Q1 ∧ Q2 else Q1 ∗ Q2.
Instance into_and_sep p P Q : IntoAnd p (P ∗ Q) P Q.
Instance into_and_and P Q : IntoAnd true (P ∧ Q) P Q.
Instance into_and_later p P Q1 Q2 : IntoAnd p P Q1 Q2 → IntoAnd p (▷ P) (▷ Q1) (▷ Q2).
Instance into_and_mapsto l q v : IntoAnd false (l ↦{q} v) (l ↦{q/2} v) (l ↦{q/2} v).

Lemma tac_and_destruct Δ Δ' i p j1 j2 P P1 P2 Q :
  envs_lookup i Δ = Some (p, P) →
  IntoAnd p P P1 P2 →
  envs_simple_replace i p (Esnoc (Esnoc Enil j1 P1) j2 P2) Δ = Some Δ' →
  (Δ' ⊢ Q) → Δ ⊢ Q.
```

IPM in summary

- Contexts are deeply embedded
- Context manipulation is done via **computational reflection**
- IPM tactics are just Coq lemmas
- **Type classes** are used to make the tactics more general
- **Ltac** is used to provide an end-user syntax and error reporting

IPM in summary

- Contexts are deeply embedded
- Context manipulation is done via **computational reflection**
- IPM tactics are just Coq lemmas
- **Type classes** are used to make the tactics more general
- **Ltac** is used to provide an end-user syntax and error reporting

These ideas are hopefully applicable to other object logics

In the paper and Coq formalization

- Detailed description of the implementation
- Verification of concurrent algorithms using IPM
- Formalization of unary and binary logical relations
- Proving logical refinements

Shows that IPM scales

Interactive Proofs in Higher-Order Concurrent Separation Logic

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Abstract

When using a proof assistant to reason in an embedded logic – like separation logic – one cannot benefit from the proof contexts and basic tactics of the proof assistant. This results in proofs that are at a too low level of abstraction because they are cluttered with bookkeeping code related to manipulating the object logic.

In this paper, we introduce a so-called *proof mode* that extends the Coq proof assistant with (spatial and non-spatial) named proof contexts for the object logic. We show that thanks to these contexts we can implement high-level tactics for introduction and elimination of the connectives of the object logic, and thereby make reasoning in the embedded logic as seamless as reasoning in the meta logic of

instance, they include separating conjunction of separation logic for reasoning about mutable data structures, invariants for reasoning about sharing, guarded recursion for reasoning about various forms of recursion, and higher-order quantification for giving generic modular specifications to libraries.

Due to these built-in features, modern program logics are *very different* from the logics of general purpose proof assistants. Therefore, to use a proof assistant to formalize reasoning in a program logic, one needs to represent the program logic in that proof assistant, and then, to benefit from the built-in features of the program logic, use the proof assistant to reason *in* the embedded logic.

Reasoning in an embedded logic using a proof assistant tradition-

Thank you!

Want a 'proof mode' for another logic, talk to us!

Download Iris at `http://iris-project.org/`

Talks about Iris this week:

- Wed 15:35 @ POPL: Krogh-Jespersen, *Svendsen* and Birkedal
  A Relational Model of Types-and-Effects in Higher-Order Concurrent Separation Logic
- Sat 9:00 @ CoqPL: *Krebbers*
  Demonstration of the Iris separation logic in Coq
- Sat 10:30 @ CoqPL: *Timany*, Krebbers and Birkedal
  Logical Relations in Iris

Coq wish list

- Data types in Ltac
- Side-effecting tactics that can return a value
- More expressive parsing mechanism of tactic notations
- Exception handling in Ltac to enable better error message generation
- Opt-out from backtracking Ltac semantics